SEMESTER END EXAMINATION, APRIL-MAY, 2025

Course Name: - B.Tech. **Semester:- IV**

Paper Name: - Engineering Mathematics-IV **Paper Code:- TBS-402**

Time - 3 Hrs + 20 minutes per hour extra time for V.I. & examinees with writer. **Max Marks-70**

Additional 30 Minutes for Mid-Test.

Instructions:

- The question paper consists of three sections namely A, B, C. All sections are compulsory.
- Section A- Each question carries **3 mark. All questions are compulsory**.
- Section B- Answer any **5 out of 7** given questions. Each question carries **7 marks**.
- Section C- Answer any **2 out of 3** given questions. Each question carries **10 marks**.
- Section D- Each question carries **02 mark**. All questions are compulsory.

Section – A (खण्ड–अ)

Objective Questions(वस्तुनिष्ठ प्रश्न)

1. Answer all the following questions. 5x3 =15

निम्नलिखित सभी प्रश्न अनिवार्य हैं।

i) If function f(z) is analytic at a point z_0 then

a) f(z) is differentiable only at z_0

b) f(z) is differentiable at every point in some nbd of z_0

c) f(z) is differentiable not only at z_0 but also at every point of some nbd of z_0.

d) f(z) is differentiable at every point in some nbd of z_0 except z_0.

ii) If $f(z) = u + iv$ is analytic then which of the following is correct?

a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

b) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} = \frac{\partial v}{\partial y}$

c) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} = -\frac{\partial v}{\partial y}$

d) None of the above

iii) Convergence rate of Newton's Raphson Method is

a) 1

b) -1

c) -2

d) 2

iv) The radius of convergence of an infinite series is given by

a) $\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

b) $\frac{1}{R} = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}$

c) Both a and b

d) None of the above

v) The mean of Binomial Distribution is given by

a) $\frac{n}{p}$

b) np

c) npq

d) $\frac{np}{q}$

Section – B (खण्ड–ब)

Short Answer Questions (लघुउत्तरीय प्रश्न)

2. Answer any five of the following questions. **5x7=35**

निम्नलिखित में से किन्हीं पाँच प्रश्नों के उत्तर दें।

i. a) Show that if a function f(z) differentiable then is continuous but converse in not true.

b) Test the analyticity of $f(z) = \sin z$ and hence deduce that $\frac{d}{dz}(\sin z) = \cos z$.

ii. a) Find the radius of convergence of function $f(z) = \sum_{n=0}^{\infty} \frac{n^n z^n}{n!}$.

b) Evaluate $\oint \frac{e^z}{z^2+1} dz$ over the circular path $|z| = 1$

iii. a) Expand $\frac{1}{z^2-3z+2}$ in the region (a) $|z| < 1$ (b) $|z| > 2$

b) If the probability of defective bolt is 0.1 find (a) the mean (b) the standard deviation for the distribution of bolts in a total of 400.

iv. a) If $u = \frac{1}{2}\log(x^2 + y^2)$ show that its harmonic find its conjugate harmonic function v. Also find $f(z)$.

b) Calculate the correlation coefficient between the following data.

X	5	9	13	17	21
Y	12	20	25	33	35

v. a) Evaluate the following integral using residue theorem $\oint \frac{1+z}{z(2-z)} dz$ over the circle $|z| = 1$.

b) An urn contains 10 black and 10 white balls. Find the probability of getting two balls of same colour.

vi. Prove the following

a) $\mu^2 = 1 + \frac{1}{4}\delta^2$

b) $D = \frac{1}{h}[\Delta - \frac{1}{2}\Delta^2 + \frac{1}{3}\Delta^3 + \cdots \ldots]$.

vii. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$ by using residue theorem.

b) Find the probability of getting 4 heads in 6 tosses of a fair coin.

Section – C (खण्ड–स)

Descriptive Questions (विवरणात्मक प्रश्न)

3. Answer any two of the following question. **2x10=20**

निम्नलिखित में से किन्हीं दो प्रश्नों के उत्तर दें।

i) a) Find the smallest positive root of the equation $x^3 - 2x + 0.5 = 0$ using Newton Raphson method.

b) Find the first and second derivative from the following table by Lagrange's method.

X	1	3	-4
Y	3	-5	4

ii) a) Using Lagrange's Interpolation formula find the values of Y corresponding to x=10

x	5	6	9	11
y	12	13	14	16

b) If θ be the angle between two regression lines in the case of two variables x and y show that $\tan\theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2+\sigma_y^2}$

iii) a) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson one-third rule and three-eight rule dividing into six equal parts

b) Using Euler's method solve the differential equation in six step: $\frac{dy}{dx} = x + y. y(0) = 0, h = 0.2$

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Mid-Test

Course Name: - B.Tech. **Semester:- IV**

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Time - 30 minutes. Max Marks-20

All questions are compulsory. 2×10=20

सभी प्रश्न अनिवार्य हैं।

Objective Questions.

बहुविकल्पीय प्रश्न।

1. The $f(x,y)$ defined by $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ when $(x,y) \neq 0$; f(0,0)=0 is
   a) Differentiable at (0,0)
   b) Such that $f_x(0,0)$ and $f_y(0,0)$ do not exist
   c) Not continuous at (0,0)
   d) Continuous but not differentiable.

2. The analytic function $f(z) = u + iv$ of which the real part is $u = e^x(x\cos y - y\sin y)$is
   a) $e^z + c$
   b) $ze^{-z} + c$
   c) $ze^z + c$
   d) $e^{-z} + c$

3. The point at which a function not analytic is called
   a) Multiple point
   b) Singular point
   c) node
   d) cusp

4. The residue of a function can be evaluated by only if the pole is an isolated singularity.
   a) True
   b) False
   c) Partially false
   d) None of these

5. In tossing of a fair die the probability of getting an odd number or a number less than 4 is
   a) 2
   b) ½
   c) 2/3
   d) 3/4

6. there are q persons sitting in a row two of them are selected at random the probability that two selected persons are not together is
   a) $\frac{2}{q}$
   b) $1-\frac{2}{q}$
   c) $\frac{q(q-1)}{(q+1)(q+2)}$
   d) *none of these*

7. The binomial distribution is given by
   a) $p(r) = {}^{n}C_{r}\, p^{r} q^{r}$
   b) $p(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$
   c) $p(r) = {}^{n}C_{r} \frac{p^{r}}{q^{r}} q^{r-n}$
   d) *none of these*

8. S.D of Poisson's Distribution is given by
   a) m
   b) 1/m
   c) $m^{1/2}$
   d) none of these

9. Successive approximation for Newton Raphson method is given by
   a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
   b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$
   c) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$
   d) $x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)}$

10. Which one is correct
   a) $\delta = E^{1/2} + E^{-\frac{1}{2}}$
   b) $\mu = \frac{1}{2}\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right)$
   c) $\Delta = E + 1$
   d) none of these

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